Lab L: Critical Points, Inflection Points, and fsolve

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Overview
The analysis of a function via calculus involves solving a variety of equations: $f'(x) = 0$ for critical points, $f''(x) = 0$ for possible inflection points. In many cases it is impossible to find exact solutions to these equations. Maple's **fsolve** command will be used to find approximate solutions to equations.

Maple Essentials

Command	Description
fsolve	Similar to **solve**. Returns a floating-point approximation. (Returns a decimal instead of an exact value.)

Preparation
Review the First and Second Derivative Tests.

Activities

- Log in and start a Maple session.
- Type **with(plots):** at the top of your worksheet. This will allow us to plot points and use the display command.
- **Example 1:** Find approximations to all solutions to $x^3 - 5x = -1$.
  (1) The first step is to rewrite the problem as a root-finding problem. That is, $x^3 - 5x + 1 = 0$.
  (2) We then graph $F(x) = x^3 - 5x + 1$ to get an approximation of the solutions.
  > plot(x^3 - 5*x + 1, x=-5..5);
  (3) Using fsolve:
    (a) For this particular graph, the first solution is between x=-3 and x=-1.
    (b) Enter the following line of code:
    > fsolve(x^3 - 5*x + 1 = 0, x=-3..-1);
    (c) You should get -2.330058740.
  (4) Get estimates for the other two solutions.
- **Example 2:** Find and plot the cubic polynomial that has a relative maximum at (-1,2) and a relative minimum at (3,-2).
  (1) We begin with a generic cubic polynomial and its derivative.
  > f:= a*x^3 + b*x^2 + c*x + d ;
  > DyDx:= diff(f, x);
  (2) We get two equations from the points (-1,2) and (3,-2).
  > eq1:= eval(f, x=-1) = 2;
  > eq2:= eval(f, x=3) = -2;

(3) We get two more equations from $f'(x) = 0$ at relative extrema.
> eq3:= eval(DyDx, x=-1) = 0;
> eq4:= eval(DyDx, x=3) = 0;

(4) We now solve the system of 4 equations and 4 unknowns. We will use the **fsolve** command to get numeric approximations instead of fractions.
> values:= fsolve({eq1,eq2,eq3,eq4}, {a,b,c,d});

(5) We find the cubic by plugging in these values into f.
> F:= eval(f, values);

(6) We then plot the cubic and the points to verify that we have the correct cubic.
> P1:= plot(F, x=-5..5, y=-5..5):
> P2:= pointplot([-1,2], symbolsize=10):
> P3:= pointplot([3,-2], symbolsize=10):
> display([P1,P2,P3]);

(7) Notice that the graph has relative extrema at (-1,2) and (3,-2) as desired.

- **Example 3:** Find the intervals over which the function $f(x) = x^{\sin(x)}$ is increasing, decreasing, concave up, and concave down on [0,6].
  (1) We begin by identifying the critical points. Use Maple to find the derivative and assign it to df.
  (2) Graph df and use the **fsolve** command to find the zeros.
  (3) The function $f(x)$ is increasing where df is positive (above the x-axis) and decreasing where df is negative (below the x-axis).
  (4) We then identify possible inflection points. Use Maple to find the second derivative and assign it to ddf.
  (5) Graph ddf and use the **fsolve** command to find the zeros.
  (6) The function $f(x)$ is concave up where ddf is positive (above the x-axis) and concave down where ddf is negative (below the x-axis).
  (7) Finally, graph $f(x)$ over [0,6] and make sure your answers make sense.

Assignment

- Your assignment for this week is to complete **Project 3**. You should prepare a neat and complete project report. All projects are due at the *beginning* of next week's lab.
- Next week's lab will be **Hour Quiz 3**. You should study all material since the last hour quiz.