Lab N: Definite Integrals and Riemann Sums

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Overview
This lab will help to develop your understanding of the definite integral as defined via Riemann sums and as computed via the Fundamental Theorem of Calculus.

Maple Essentials

- The *Riemann Sums* tutor is started from the Maple 9.5 user interface under the tools menu:
  Tools → **Tutors** → **Calculus - Single Variable** → **Riemann Sums** ...
- The new Maple command introduced in this lab is **int** used for definite and indefinite integrals.

Preparation
Review the definition of area under a curve and approximations of area and the Fundamental Theorem of Calculus. In particular, you should be able to explain the symbols and meaning of the following two equations:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

$$\int_a^b f(x)dx = F(b) - F(a) \quad \text{where } F \text{ is the antiderivative of } f$$

Activities

(1) Log in and start a Maple session.

(2) **Example 1:** Use the *Riemann Sums* tutor to approximate $\int_2^{10} \frac{1}{x}dx$ with the Riemann sum $\sum_{k=1}^{4} f(x_k^*)\Delta x$ where:
- x_k^* is the left endpoint of each subinterval
- x_k^* is the right endpoint of each subinterval
- x_k^* is the midpoint of each subinterval

Then increase the number of subintervals and describe what happens to your approximation.
   (a) Launch the *Riemann Sums* tutor.
   (b) Plug in $f(x) = 1/x$, $a = 2$, $b = 10$, and $n = 4$.
   (c) Click on left and press **Display**. Notice how each rectangle has the height of the left endpoint's function value.
   (d) Repeat for right and midpoint.
   (e) Input other values for n, say 8, 16, 32, 64, 128, etc, clicking **Display** each time. What happens to your approximation?

(3) **Example 2:** Use Maple to evaluate the following definite integrals:

a. $\int_0^{\pi/2} \cos(x)dx$

b. $\int_2^6 x^3 dx$

c. $\int_{-1}^3 e^{-x} dx$

d. $\int_0^4 \frac{x}{x+1} dx$

e. $\int_0^4 \frac{x}{x^3+1} dx$

f. $\int_0^{3\pi/2} \cos(x)dx$

g. $\int_0^5 \sqrt{x} dx$

h. $\int_{-1}^3 xe^{-x} dx$

i. $\int_0^4 \frac{x}{x^2+1} dx$

j. $\int_0^4 \frac{x}{x^4+1} dx$

(4) We will walk through the first example together. Input the following lines of code.
> f:= cos(x);
> int(f, x=0..Pi/2);
Ask your TA how to use the **Expression** palette if you have forgotten.

Assignment

This is the last lab of the semester, and you have already completed three projects and three quizzes. Congratulations! There is no assignment this week.