Lab H: More Mathematical Models

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Overview

There are three objectives in this lab

- understand the mathematical reasoning associated with a real-world example,
- use Maple to solve a system of equations, and
- use Maple to graph a piecewise-defined function.

Preparation

Review properties of the first derivative. Review higher order derivatives.

The Problem

Suppose you are asked to design the first ascent and drop for a new roller coaster. By studying photographs of your favorite coasters, you decide to make the slope of ascent 0.8 and the slope of the drop -1.6. You decide to connect these two straight stretches $y = L_1(x)$ and $y = L_2(x)$ with part of a parabola $y = f(x) = ax^2 + bx + c$, where x and $f(x)$ are measured in feet. For the track to be smooth there can't be abrupt changes in direction, so you want the linear segments L_1 and L_2 to be tangent to the parabola at the transition points P and Q. (See the figure.) To simplify the equations, you decide to place the origin at P.

Answer the following questions.

(1) Suppose that the horizontal distance between P and Q is 100ft. Write equations in a, b, and c that will ensure the track is smooth at transition points.
(2) Solve the linear equations in part (1) for a, b and c to find a formula for $f(x)$.
(3) Plot L_1, f, and L_2 to verify graphically that the transitions are smooth.
(4) Find the difference in elevation between P and Q.

Source: Stewart, James. *Calculus: Early Transcendentals Single Variable.* Thomson Brooks/Cole. 2003.

Solving the Problem

(1) Log in and start a Maple session.
(2) Type **with(plots):** and press enter. This will let us use the display command later.
(3) Define f as $ax^2 + bx + c$.
(4) Define Df as the derivative of $f(x)$.
(5) Question (1).
  (a) If the point P is at the origin, we know that $f(0) = 0$. This will be our first equation.
  (b) If the line L_1 with slope 0.8 is tangent to $f(x)$ at $x = 0$, we know that $f'(0) = 0.8$. This will be our second equation.
  (c) If the line L_2 with slope -1.6 is tangent to $f(x)$ at $x = 100$, we know that $f'(100) = -1.6$. This will be our third equation.
(6) Question (2).
  (a) We solve the equations from question (1) using the **solve** command. We assign the solutions to *values*.
  (b) We then plug these values in to find $f(x)$. Remember to re-assign the equation to the variable f.
(7) Question (3).
  (a) Before we can plot, we must find equations for L_1 and L_2.
  (b) L_1 is a line through $(0, 0)$ with a slope of 0.8.
  (c) L_2 is a line through $(100, f(100))$ with a slope of -1.6.
  (d) We want to plot L_1 from -20 to 0. This will be our first plot command. Remember to use : not ;.
  (e) We want to plot $f(x)$ from 0 to 100. This will be our second plot command. We will use a different **linestyle** and **color** to make sure we can distinguish the sections of the graph.
  (f) We want to plot L_2 from 100 to 120. This will be our third plot command.
  (g) We use the display command to graph the piecewise-defined function.
(8) Question (4).
  (a) We know that Q is the point $(100, f(100))$. So we find $f(100)$.
  (b) Since P is the points $(0, 0)$, we easily calculate the difference in elevation.
(9) Remember to log out.

Assignment

Your assignment for this week is to complete Project 2. You should prepare a neat and complete project report. All projects are due at the *beginning* of next week's lab.