

Integration Methods II: Partial Fractions and Trig Substitutions

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Overview

The objective of this lab is to use Maple to practice integration skills using partial fraction decomposition and trigonometric substitutions.

Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description
<code>convert(f(x), parfrac, x);</code> Example: <code>convert(x/((x^2+1)*(x-2)^3), parfrac, x);</code>	Converts a rational function $f(x)$ into its partial fraction form.
<code>completesquare(f(x), x);</code> Example: <code>completesquare(x/(x^2+2*x+2), x);</code> Note: You need to load the <i>student</i> package before using this command.	Completes the square to rewrite a quadratic expression.

- The *Partial Fractions: General Decomposition* maplet is available from the course website:

<http://www.math.sc.edu/calclab/142L-F09/labs> → [Partial Fractions: General Decomposition](#)

This maplet is designed to help you practice partial fraction decomposition. Click on **New Function** to have the maplet generate a problem for you to practice. Then add terms one at a time until you have the general form for the decomposition. Once you are confident, click on the **Include Completing the Square** box to make the decomposition more difficult.

- The *Partial Fractions: Evaluating the Integral* maplet is available from the course website:

<http://www.math.sc.edu/calclab/142L-F09/labs> → [Partial Fractions: Evaluating the Integral](#)

This maplet is designed to help you practice evaluating the integral after using partial fraction decomposition. Click on **Modify or Make Your Own Problem** to enter an integral. Click on **Show Partial Fraction Expansion**. Then evaluate the expanded integral and check your work.

- The *Trig Substitution* maplet is available from the course website:

<http://www.math.sc.edu/calclab/142L-F09/labs> → [Trig Substitution](#)

This maplet is designed to help you practice the steps involved in integration using trigonometric substitutions. Click on **New Integral** to have the maplet generate a problem for you to practice. Next, identify the type of trigonometric substitution and follow the steps to solve the problem. (You will need a pencil and paper to do all the steps yourself.) You can click **Hint** or **Show** if you need help or want to skip a step.

Related course material

§8.4 and §8.5 of the textbook.

Activities

1. Launch the *Partial Fractions: General Decomposition* maplet from the course web page and do a few practice problems.
2. Follow the steps below to integrate the following rational functions:

(a) $f_1(x) = \frac{1}{x^2 - 6x - 7}$. (Page 543, ex. 10)

(b) $f_2(x) = \frac{x^5 - 4x^3 + 1}{x^3 - 4x}$. (Page 544, ex. 20)

(c) $f_3(x) = \frac{x^2}{(x+1)^3}$. (Page 544, ex. 25)

(d) $f_4(x) = \frac{x^3 + x^2 + x + 2}{(x^2 + 1)(x^2 + 2)}$. (Page 544, ex. 30)

(e) $f_5(x) = \frac{x^4 + 6x^3 + 10x^2 + x}{x^2 + 6x + 10}$. (Page 544, ex. 32)

(f) $f_6(x) = \frac{x^2 + 1}{(x^2 + 2x + 3)^2}$. (Page 544, ex. 37)

Steps:

The following steps would be used to integrate $f_1(x)$.

- (a) Load the maple `student` package.
`> with(student):`
 - (b) Assign the function using the arrow notation.
`> f1 := x -> 1/(x^2-6*x-7);`
 - (c) Write down the general form for the partial fraction decomposition of the function by hand. Then use Maple to complete the expansion and compare it to your general form.
`> convert(f1(x), parfrac, x);`
 - (d) Use a pencil and paper to integrate the function. Then use Maple to complete the integration and check your answer.
`> int(f1(x),x);`
 - (e) You can also use the *Partial Fractions: Evaluating the Integral* maplet to check your integration.
3. Launch the *Trig Substitution* maplet from the course web page and do a few practice problems. Remember, you will need a pencil and paper to complete all the steps by yourself.

Assignment

Complete the lab activities and do the following problems with the help of Maple: 38 and 39 on page 544. (Show the partial fraction decomposition for both rational functions.)